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GRT - well proven and also incomplete. Further arguments

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Abstract

There are two contradictory formulas of the total energy of a particle resting in the gravitational field [1]-[3]. From the formulas of radial free fall one gets:

(2) $$E_G = mc^2\left(1 - \frac{2GM}{c^2 r}\right)^{1/2}$$

On the other side, there is the equivalence principle. A particle resting in its local inertial system (i.e. the freely falling particle) has a total energy equal to its rest mass:

(3) $$E_G = mc^2$$

Both of the formulas contradict each other since it doesn't matter whether the particle is at rest in the gravitational field $t = 0, v = 0, b = 0$ or becomes accelerated $t = 0, v = 0, b \neq 0$ [3]. Lorentz interpretation (LI) of GRT solves this contradiction with the assumption that standard clocks in gravitational fields run slower by a factor

(6) $$\left(1 - \frac{2GM}{c^2 r}\right)^{1/2}$$

The talk proves this assumption in a larger context using the energy relation of arbitrarily moving particles.
These considerations are not really difficult. In spite of this, they become 'rejected' by arguments which contradict each other [2], [3].

This contribution is a translation of those parts of the talk which are relevant to Lorentz interpretation of GRT (LI of GRT).

Introduction and educational arguments

(not included)

Main results of „GRT - well proven and also incomplete?"

This talk expands the ideas of [3] from particles resting in the gravitational field to particles moving arbitrarily in the field. The two contradictious energy formulas for the resting particle are the formulas (2) and (3) of [3].

(2) $$E_G = mc^2\left(1 - \frac{2GM}{c^2 r}\right)^{1/2}$$

and

(3) $$E_G = mc^2$$

(2) is at least qualitatively correct since removing the particle from the gravitational field needs energy. Doing this the total energy E_G of the particle becomes mc^2 and therefore within the gravitational field E_G has to be lower. On the other side, there is the equivalence principle. A particle resting in its local inertial system (i. e. the free falling particle) has a total energy equal to its rest mass which is formula (3)
Formulas (2) and (3) contradict each other. Certainly, they belong to different reference systems with one of them being accelerated, in fact. But: At time point $t = 0$ the free falling particle is also a resting one within the *r,t*-reference system since its velocity $v = 0$. Only its acceleration $b \neq 0$. Special theory of relativity is

applicable and therefore the free falling particle at $t=0$ and an always resting particle at the same place possess identical total energies (3), see [3].
Another argument for contradiction: Check formulas (2) and (3) by experiment. The measuring instruments resting at the same place as the particle does have only one measuring result and not two different ones. Only one of the two formulas can be proved experimentally.
Let us choose some intellectually simple measuring procedure. Transfer an antiparticle to the resting particle and perform the measurement of their annihilation frequency. One gets:

(4)
$$\begin{aligned} E_{G,measured} &= mc^2 \\ &= h\nu_{\tau,measured} \end{aligned}$$

$\nu_{\tau,measured}$: Annihilation frequency, measured by a clock resting in the gravitational field

τ : proper time of a clock resting in the gravitational field (τ -clock, standard clock)

Every complete clock measures one frequency and not different two ones at the same time. The experimental result has to confirm formula (3) otherwise the equivalence principle would be wrong which is obsolete since the equivalence principle is well proven.
On the other side, formula (2) is correct. One can see it again by series expansion of (2):

(5)
$$E_G = mc^2 - \frac{GmM}{r} \pm \ldots$$

The second term describes the negative gravitational energy. Approximately formula (2) becomes the rest mass minus Newtonian gravitational energy. Therefore, formula (2) meets the Newtonian limit but formula (3) does not.
The attempts of classical GRT to solve the contradiction of (2) and (3) are examined in [3]. Here a further argument is given supporting LI of GRT.
Lorentz-interpretation (LI) of GRT solves this contradiction with the assumption that standard clocks in gravitational fields run slower by a factor

(6)
$$\frac{d\tau}{dt} = \left(1 - \frac{2GM}{c^2 r}\right)^{1/2}$$

derivable from SM (Schwarzschild metric). The measuring result of formula (3) corrected by this factor results in formula (2).

General case

Do these considerations remain correct for the general case?
The general case means that the particle m is moving like a planet around the gravitational center (sun or star) and is not constantly at rest. The formula for total energy per unit mass k becomes

(7)
$$k^2\left(1-\frac{2GM}{c^2 r}\right)^{-1} - \frac{\dot{r}^2}{c^2}\left(1-\frac{2GM}{c^2 r}\right)^{-1} - \frac{r^2\dot{\varphi}^2}{c^2} = 1$$

as is shown by [4], [5]. Using

(8)
$$E_G = mc^2 k$$

one gets

(9)
$$E_G^2 = \left(mc^2\right)^2\left(\left(1+\frac{r^2\dot{\varphi}^2}{c^2}\right)\left(1-\frac{2GM}{c^2 r}\right)+\frac{\dot{r}^2}{c^2}\right)$$

This is the generalisation of formula (2) and is derived from the general formula of free fall. Generalisation of formula (3) results from the equivalence principle:

(10) $$E_G = mc^2 \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

As in [3] the equivalence principle is reworded: “For **measurements** within gravitational fields the **measuring** results within local inertial systems are predicted by special relativity.”

Concerning our application this means: The measurement of E_G with measuring instruments resting in the gravitational field yields (10). This is no contradiction to (9) any longer if one can assume that measuring instruments become modified by gravitational fields.

The measurement of E_G remains similar to [3]. Particle m and antiparticle $\overline{m}$ hit one another with opposite velocities v and $-v$. The measured annihilation frequency gives the total energy E_G. Both of the formulas (2) and (3) may differ with a factor of (6) only. Only in this case LI of GRT remains able to explain the difference between formulas (9) and (10). LI of GRT claims that measuring rods and clocks become changed by gravitational fields but frequency measurements need clocks alone, only their change in velocity (6) has to explain (9) and (10).

(9) and (10) contradict each other at least because their limiting cases $v = 0, \dot{\varphi} = 0$, formulas (2) and (3), do.

The assertion is

(11) E_G der Formel {9} $=$ E_G der Formel {10} x Formel {6}

$\dot{\varphi}$ in formula (9) is written as $\overset{*}{\varphi}$, $*$ means proper time of a clock moving together with particle m_0. The proper time of a clock resting in the gravitational field is signed by point “.”. So $\dot{\varphi}$ is the angular velocity of particle m_0 measured with a clock resting in the gravitational field, $\overset{\bullet}{\varphi}$ is the angular velocity of particle m measured with a clock resting near the particle m. Similar $\dot{v}, \overset{*}{v}$.

So

(12) $$\frac{d\varphi}{d\tau^*} = \frac{d\varphi}{d\tau}\frac{d\tau}{d\tau^*}$$

or

(13) $$\overset{*}{\varphi} = \dot{\varphi}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

and

(14) $$\overset{*}{r} = \dot{r}\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}.$$

Insert formulas (13) – (14) into (9) and you will get:

(15) $$E_G{}^2 = \left(mc^2\right)^2\left(\left(1+\frac{r^2\dot{\varphi}^2}{c^2}\frac{1}{\left(1-\frac{v^2}{c^2}\right)}\right)\left(1-\frac{2GM}{c^2 r}\right)+\frac{\dot{r}^2}{c^2}\frac{1}{\left(1-\frac{v^2}{c^2}\right)}\right)$$

On account of

(16) $$\left(r\dot{\varphi}\right)^2 = v_{\tan}{}^2$$

(17) $$dr_{rad} = ds\left(1-\frac{2GM}{c^2 r}\right)^{\frac{1}{2}}$$

(17) is formula (20.3) in [1] and results in:

(18) $$\dot{r}^2 = v_{rad}{}^2\left(1-\frac{2GM}{c^2 r}\right)$$

(19) $$v_{rad}{}^2 + v_{\tan}{}^2 = v^2$$

On account of {16}-{19}one gets

(20) $$E_G{}^2 = \left(mc^2\right)^2 \frac{1}{\left(1-\frac{v^2}{c^2}\right)}\left(1-\frac{2GM}{c^2 r}\right)$$

This is what we want:

(11) E_G der Formel {9} = E_G der Formel {10} x Formel {6}

Summary

There is a contradiction between the energy formula of a free falling particle derived from the equations of free fall (9) and from the equivalence principle (10). LI of GRT solves this contradiction by the assumption that clocks are slowed down in gravitational fields (6) and by a minor restriction of the equivalence principle: "For **measurements** within gravitational fields the **measuring** results within local inertial systems are predicted by special relativity." ...

Literature

[1] J. Brandes, J. Czerniawski: *Spezielle und Allgemeine Relativitätstheorie für Physiker und Philosophen - Einstein- und Lorentz-Interpretation, Paradoxien, Raum und Zeit, Experimente,* VRI 2010, 4. erweiterte Auflage, 404 Seiten, 100 Abbildungen, ISBN 978-3-930879-08-3 Näheres (Preis, Inhaltsverzeichnis etc.): http://www.buchhandel.de/ oder http://www.amazon.de/

[2] „Ist die klassische Allgemeine Relativitätstheorie unvollständig?", see: http://www.grt-li.de/

[3] „GRT - well proven and also incomplete?", see: http://www.grt-li.de/

[4] Dragon, Norbert (2012): *Geometrie der Relativitätstheorie.* http://www.itp.uni-hannover.de/~dragon. pdf-file auf der Homepage des Autors. Gleichung (6.21) und (6.24)

[5] d´Inverno, R. (Hrsg. G. Schäfer, Jena), *Einführung in die Relativitätstheorie.* Weinheim, New York, Basel, Cambridge, Tokio: VCH 1995. Gleichung (15.23)